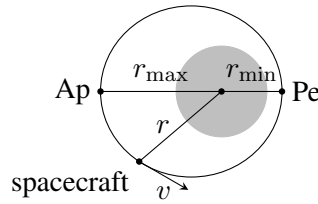


Orbital maneuvering

1 Orbital speed

The following formula describes the speed of a spacecraft orbiting a planet or moon:



$$v^2 = MG \left(\frac{2}{r} - \frac{2}{r_{\min} + r_{\max}} \right), \quad (1)$$

where M is the mass of the planet in kilograms, G is the gravitational constant ($6.673\,84 \times 10^{-11}$), $r_{\min}$ is the distance from the periapsis to the center of the planet in meters, $r_{\max}$ is the distance from the apoapsis to the center of the planet in meters, r is the distance from the current position of the craft to the center of the planet in meters and v is the velocity of the craft in meters per second.

Because MG is generally known at a better precision than M and G separately, the so called gravitational parameter $\mu = MG$ is usually listed directly in planetary almanacs, such as the KSP wiki. Also, because the average of the periapsis and apoapsis, called the semi-major-axis, is used in a lot of formulae, we denote $a = \frac{r_{\min} + r_{\max}}{2}$. Substituting μ and a in the formula above gives:

$$v^2 = \mu \left(\frac{2}{r} - \frac{1}{a} \right). \quad (2)$$

Finally, as we're usually interested in the velocity v and not v^2 , we take the root:

$$v = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a} \right)}. \quad (3)$$

Example: Bob is in orbit around Kerbin. His instruments show that his periapsis is at 80 km, his apoapsis is at 160 km and his current altitude is 119.253 km. Note that these instruments tell us the altitudes relative to the surface of Kerbin and the formula wants them as distances to the center of Kerbin. We can read on the KSP wiki that the radius of Kerbin is 600 km. To calculate the semi-major-axis of Bob's orbit, we add the radius of Kerbin to Bob's periapsis and apoapsis:

$$a = \frac{r_{\min} + r_{\max}}{2} = \frac{(80000 + 600000) + (160000 + 600000)}{2} = 720000 \text{ m} = 720 \text{ km}.$$

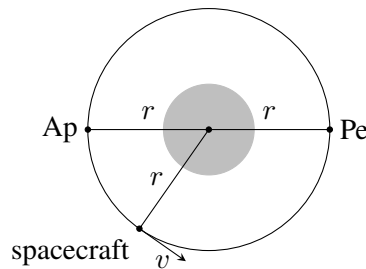
We also read in the wiki that the gravitational parameter μ of Kerbin is 3.5316×10^{12} . The current velocity of Bob's craft should be:

$$v = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a} \right)} = \sqrt{3.5316 \times 10^{12} \left(\frac{2}{119253 + 600000} - \frac{1}{720000} \right)} \approx 2217 \text{ m/s}.$$

You can check in KSP to verify.

2 Speed in circular orbits

When a craft is in a circular orbit, the periapsis and apoapsis of the orbit, as well as the current altitude of the craft, are all the same. This simplifies the formula for calculating the speed of the craft:



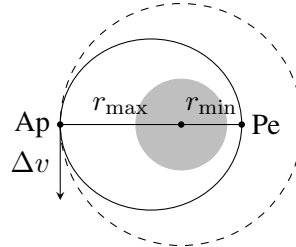
$$v^2 = MG \left(\frac{2}{r} - \frac{2}{r_{\min} + r_{\max}} \right), \quad (4)$$

$$v^2 = MG \left(\frac{2}{r} - \frac{2}{r + r} \right), \quad (5)$$

$$v^2 = MG \left(\frac{1}{r} \right), \quad (6)$$

$$v = \sqrt{\frac{\mu}{r}}. \quad (7)$$

Example: Bob decides to circularize his orbit around Kerbin. Currently his periapsis is 80 km and his apoapsis is 160 km. He decides to burn at apoapsis to raise his periapsis.



How much Δv should he burn for? To answer this, we need to know how fast he's going when he reaches apoapsis and how fast he *should* be going in order to obtain a circular orbit. His speed at apoapsis will be:

$$v_{Ap} = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a} \right)} = \sqrt{3.5316 \times 10^{12} \left(\frac{2}{160000 + 600000} - \frac{1}{720000} \right)} \approx 2095 \text{ m/s},$$

and to obtain a circular orbit, he *should* be going:

$$v_{\text{circ}} = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{\mu}{160000 + 600000}} \approx 2156 \text{ m/s}.$$

So his burn must increase his speed with $v_{\text{circ}} - v_{Ap} = 2156 - 2095 = 61 \text{ m/s}$.

3 Orbital period

The formula for the orbital period of a spacecraft orbiting a planet is:

$$T = 2\pi \sqrt{\frac{a^3}{\mu}},$$

where a is the semi-major-axis (see [section 1](#)), μ is the gravitational parameter of the planet (see also [section 1](#)), and T is the time it takes to complete one orbit in seconds.

Example: Mission control wants to put a satellite into orbit around Kerbin, so that it always floats exactly above the KSP; a so called synchronous orbit. In this special circular orbit, the satellite takes exactly as long to complete an orbit as Kerbin does to rotate around her axis, or in other words, the

orbital period of the satellite is exactly one Kerbin day. How fast should the satellite be going and at which altitude should it be flying? The KSP wiki tells us one Kerbin day is 21600 s. If the orbital period of the satellite should be equal to that, the semi-major-axis of its orbit should be:

$$\begin{aligned}
 T &= 2\pi\sqrt{\frac{a^3}{\mu}}, \\
 \sqrt{\frac{a^3}{\mu}} &= \frac{T}{2\pi}, \\
 \frac{a^3}{\mu} &= \frac{T^2}{4\pi^2}, \\
 a^3 &= \frac{\mu T^2}{4\pi^2}, \\
 a &= \sqrt[3]{\frac{\mu T^2}{4\pi^2}} = \sqrt[3]{\frac{3.5316 \times 10^{12} \cdot 21600^2}{4\pi^2}} \approx 3468751 \text{ m}.
 \end{aligned}$$

Since in a circular orbit, the periapsis and apoapsis are equal, the semi-major-axis is also equal to the altitude at which the satellite will be flying. However, this is measured as the distance to the center of the planet, while generally in KSP, the altitude is measured relative to the surface of the planet. The KSP wiki tells us the radius of Kerbin is 600 km, so the altitude of a satellite in Kerbin-synchronous orbit is $3468751 - 600000 = 2868751 \text{ m} = 2868.751 \text{ km}$. The speed at which it will be flying (see [section 2](#)) is:

$$v = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{3.5316 \times 10^{12}}{3468751}} \approx 1009 \text{ m/s}.$$